## Validation Studies Of Quasilinear Theory Of Resonant Diffusion In The Ion Cyclotron Range Of Frequencies By Comparison With Exact Integration Results

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Abstract. The validity of ICRF quasilinear (QL) diffusion theory is examined in the context of the C-Mod ICRF experiment by comparison with coefficients calculated from Lorentz orbits in AORSA full-wave fields, using the DC (Diffusion Coefficient) code. The "exact" Lorentz orbit results largely agree with QL theory, although there are differences in detail. Overall conclusions are that approximation of the excited RF by a single toroidal mode leads, in the Lorentz calculation, to strong correlation pitch angle modification of the RF diffusion; this thereby modifies self-consistent radial power absorption calculated with the CQL3D Fokker-Planck code. However, inclusion of a full toroidal mode spectrum results in most, but not all, correlations ceasing to exist. Hence, modeling of ICRF power absorption using a correlation-less QL theory is reasonably accurate, even with a suitably chosen single toroidal mode. Multi-mode correlations remain, particularly for neighboring resonances such as when the trapped particles pass successively through resonance. Differences between "exact" and QL theory lead to modification of calculated, perpendicular viewing NPA spectra.

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## **INTRODUCTION**

The DC diffusion coefficient calculator numerically integrates the trajectories of ions launched from tokamak midplane points in the combined equilibrium and AORSA[1] full-wave RF fields. Particles are launched equispaced in initial gyrophase about a given gyro-center and also equispaced in toroidal length. The diffusion coefficients are obtained by averaging the resulting square of the velocity changes after one (or more) particle poloidal circuits, to obtain the ICRF bounce-averaged diffusion tensor. This is carried out for a 3D array ( $u_{\parallel}, u_{\perp}, R$ ) of initial conditions, giving the six independent RF diffusion coefficients in 3D constant-of-motion space. The method follows the formalism of Refs. [2,3]. For comparison, we have the zerobanana-width RF diffusion coefficients calculated in the AORSA code[1]. Comparison is more directly achieved in the present work by subtracting off the perpendicular guiding center drifts using a fictitious force in the Lorentz equation,  $\mathbf{F}_{\perp} = \mathbf{u}_{gc} \times \mathbf{B}$ . This removes the finite banana width effects, but leaves correlation, finite gyro-radius, and other effects. The integration of  $(32 \text{ radii}) \times (128 u_{\perp}) \times (256 u_{\parallel}) \times (8 \text{ gyro-phase}) \times (32 \text{ toroidal angle})$  starting positions (268M Lorentz orbits) is well-parallelized and takes 7 hours on 4096 cores; these global calculations are enabled by recent advances in supercomputing[4].

The DC code is similar to the MOKA code[5], but has been coupled to the CQL3D Fokker-Planck code[6] and AORSA to obtain a time-dependent, noise-free solution to the ICRF heating problem across the whole plasma width.

## **RESULTS**

Fig. 1(a-c) compares the velocity space  $D_{uu}$  diffusion coefficient calculated by DC for 1, 2, and 4 complete turns in the poloidal plane, for a symmetric 101 toroidal mode case modeling the finite length C-Mod antenna. The Fig. 1(b) coefficient for 2 turn shows significantly greater pitch angle dependence than the single turn results in 1(a); Fig. 1(c) for 4 turns shows little additional correlation effects. The close similarity of (b) and (c) support the accuracy of the code. Peaks of the Fig. 1 coefficients are 1.46(1 turn), 1.66(2 turns), 1.77(4 turns), and 0.55(AORSA QL), in accord with heuristic expectations for correlations which are reaching maximum effect. The AORSA-QL coefficients are calculated directly from the full-wave fields using quasilinear theory which incorporates spatial delta function wave-particle interactions at resonances, with no correlations[1]. All three DC coefficient radial sets show remarkable agreement in radial power absorption, shown in Fig. 2.



**FIGURE 1.** *D<sub>uu</sub>* diffusion coefficients: (a) From DC, 1 poloidal turn; (b) From DC, 2 turns; (c) From DC, 4 turns; and (d) From AORSA QL coefficient calculation. These coefficients, for C-Mod minority H, are at the same radii near the peak of the radial absorption profile, and give approximately equal power absorption.

The overall conclusion is that correlation effects are fully developed after two poloidal turns in this case where the waves are absorbed after about one half toroidal turn.

The AORSA-DC-CQL3D code suite has been stepped forward in time, and ion distributions compared with results from time-stepping the AORSA-CQL3D suite which uses only quasilinear coefficients. Figure 3, at t=4.0 msec, shows large differences in the near perpendicular velocity particles. The difference may be due to the more physically accurate model of DC which includes a full treatment of tangent resonances, breaking the usual onedimension ql diffusion[7], and/or other



FIGURE 2. Radial profile of ICRF power deposition derived from DC with 1, 2, and 4 poloidal turns, and from AORSA QL coefficients.

aspects of the Lorentz integration model. Fig. 4 shows detailed gyro-center orbits obtained in DC, giving the  $D_{uu}$  diffusion coefficient for starting location at the Figs. 1 and 3 radius, for a particular initial energy and pitch angle. The four near-tangent resonances extend through about half the bounce period, rather than being four isolated contributions as in AORSA ql calculations. These physics issues are being further investigated.



FIGURE 3. H<sup>+</sup> distributions at radius ρ=0.143 near the peak of power absorption after 4.0 msec of time evolution: (a)The classical 'rabbit' ears distribution resulting from one dimensional quasilinear diffusion. (b) Filled in trapped-particle distribution resulting from the physically comprehensive DC code model. This may be due to higher order terms beyond point support QL coefficients, due to near tangent resonance condition[7] or a Lorentz orbit effect.



**FIGURE 4.** DC orbits originating on the equatorial plane at the same location as in Figs. 1 and 3, launched with pitch angle inside the trapped-passing boundary and such that the banana tips just pass through resonance, at 0 in the central panel. AORSA would show 4 separate diffusion contributions. The eight orbits are launched equispaced in gyrophase giving the various  $u(t)-u_0$  curves in the bottom panel. <Z> is the gyro-phase averaged vertical position.

The NPA spectra resulting from the AORSA-CQL3D as compared to from AORSA-DC-CQL3D are shown in Fig. 5. The vertical perpendicular viewing chords on C-mod are particularly sensitive to the minority ion perpendicular velocity distribution, which is most affected by the enhanced pitch angle scattering calculated with DC. This is in accord with some aspects of experimental NPA results reported by Bader et al at this conference[8]. Additional calculations are being carried out to further verify and delineate the apparent discrepancy with standard quasilinear theory.



**FIGURE 5.** NPA energy spectra from (a) CQL3D using AORSA QL coefficients, and (b) using DC Lorentz orbit based coefficients, at *t*=4 msec. Results are for three vertical view cords and one horizontal chord (lowest, at low energy). The DC coefficient case has a relatively high temperature tail, and less difference between viewing chords, as a result of the more filled in perpendicular distribution as shown in Fig.3.

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