IAEA TCM, Arles, 1991 (Sept 23-

FOKKER-PLANCK FORMULATION FOR RF CURRENT DRIVE, INCLUDING WAVE DRIVEN RADIAL TRANSPORT

K. KUPFER
Département de Recherches sur la Fusion Contrôlée
Centre d'Etude de Cadarache,
13108 Saint-Paul-lez-Durance, France

ABSTRACT

A toroidal, relativistic, three-dimensional Fokker-Planck model for RF current drive is presented. The model properly accounts for the coupling of radial and velocity space dynamics, as driven by the wave-particle interaction and by collisional scattering (in the weak collisionality, or "banana regime"). The quasilinear RF diffusion tensor (in a three-dimensional constant of motion space) is cast in a form which can be evaluated from ray tracing models. The tensor elements responsible for radial transport effects are seperated into classical and neoclassical contributions. The classical contributions are largely driven by the poloidal components of the wave vectors, which are sensitive to toroidal effects as the waves propagate into the plasma. The neoclassical contributions are driven by spatially localized diffusion in energy and pitch-angle. The formulation is intended for numerical simulation of highly non-Maxwellian distribution functions, including wave driven and collisional transport effects.

1. INTRODUCTION

Externally injected waves drive toroidal current in tokamaks by appropriately distorting the electron distribution function through quasilinear diffusion. Lower-hybrid waves and fast Alfven waves induce diffusion in parallel velocity, while electron cyclotron waves induce diffusion primarily in perpendicular energy. These effects are adequately described by two-dimensional velocity space Fokker-Planck models. To model the RF current profile in experiments, the velocity space Fokker-Planck equation is solved (usually numerically) at a number of radial grid points, where the local quasilinear diffusion coefficients are obtained from a numerical treatment of the wave propagation and damping. Although such an approach can often provide an adequate description of the current profile, questions still arise concerning the effect of fast electron radial transport. These questions concern both anamolous, as well as RF driven and neoclassical transport effects.

The effect of anamolous transport has often been a concern in RF current drive and various researchers have considered the addition of a phenomenalogical radial diffusion term to the velocity space Fokker-Planck equation. It has been found that the radial diffusion of fast electrons can significantly effect the RF driven current profile (relative to the local RF absorption profile) when the radial diffusion time is on the order of the collisional slowing down time of fast electrons [1,2]. Although efforts have been made to incorporate this effect into sophisticated LHCD simulations [3,4], only recently has radial transport been included in fully three-dimensional Fokker-Planck simulations [5-7].

The effect of wave induced transport on current drive has been considered previously by several researchers. Although Antonsen and Yoshioka [8] have developed a systematic neoclassical transport theory that includes the effect of RF waves, their approach is valid for cases when the distribution function is only weakly perturbed by RF quasilinear diffusion in velocity space. Other researchers have included wave driven radial transport in Fokker-Planck models that account for strong electron tail formation, but ignore neoclassical effects [9,10]. Here we present a more complete Fokker-Planck formulation, which includes both collisional and wave driven transport effects in toroidal geometry.

The unperturbed particle orbits in a tokamak are characterized by three constants of motion, which we denote by the vector $\mathbf{I} = (I_1, I_2, I_3)$. The Fokker-Planck equation in $\mathbf{I}$ space can be written in the following form:

$$\frac{\partial}{\partial t}\mathcal{J} f_0(\mathbf{I}, t) = \frac{\partial}{\partial I_i}\mathcal{J}[D^{ij}\frac{\partial}{\partial I_j} - F^i] f_0(\mathbf{I}, t) \quad , \tag{1}$$

where a summation convention is implied and $f_0(\mathbf{I}, t)$ is the electron distribution function, averaged over the gyro-phase, bounce-phase, and toroidal angle. The notation is such that f_0 is the number of electrons in the volume element $\mathcal{J} d^3\mathbf{I}$. The Fokker-Planck coefficients D^{ij} and F^i are composed of the following contributions:

$$\begin{aligned} D^{ij} &= D_c^{ij} + D_{ql}^{ij} \\ F^i &= F_c^i + F_T^i \quad , \end{aligned} \tag{2}$$

where D_c^{ij} and F_c^i are the collisional coefficients, D_{ql}^{ij} is the quasilinear diffusion tensor due to the RF fields, and F_T^i are the coefficients which account for the toroidal electric field induced by the ohmic transformer. We take I_1 and I_2 to denote a particle's magnetic moment and the energy, whereas I_3 is considered to be a radial like coordinate, such as a particle's bounce-averaged flux surface. Although the processes associated with anamolous transport may result in a modification of the Fokker-Planck coefficients in (2), these effects are not considered here. We note, however, that a reasonable argument can be made for including anamolous transport in the coefficients D^{33} and F^3 alone, i.e. by assuming that anamolous transport is associated with a purely radial flux.

In the absence of quasilinear diffusion, Bernstein and Molvig [11] have shown that the above Fokker-Planck formulation is capable of reproducing neoclassical transport theory in the banana regime. The neoclassical bootstrap current is driven by the off-diagonal Fokker-Planck coefficients D_c^{13} and D_c^{23}, which give rise to velocity space flows in the presence of radial gradients. In RF current drive, the bootstrap effect can be modified by strong quasilinear diffusion in velocity space and by the off-diagonal Fokker-Planck coefficients due to the waves themselves, i.e. by the addition of D_{ql}^{13} and D_{ql}^{23}. Likewise, the RF fields can also modify convective radial flows (such as the neoclassical Ware pinch), which are determined by the net coefficients D^{31}, D^{32}, and F^3.

In this paper, we are concerned with the specification of the Fokker-Planck coefficients appearing in (1). The paper is organized as follows. Section 2 gives a brief review of relativistic particle orbits in a tokamak, using the Hamiltonian guiding center technique developed by Littlejohn [12,13]. The Hamiltonian approach simplifies the formal derivation of (1) and is useful in the calculation of the quasilinear diffusion tensor. The derivation of the Fokker-Planck equation, which yields the coefficients D^{ij} and F^i, is given in Section 3 and closely follows the approach of Bernstein and Molvig [11]. In Section 4, we consider the quasilinear diffusion tensor D^{ij}_{ql} in more detail; an explicit form for D^{ij}_{ql} is derived for cases when the RF field can be described by geometric optics.

2. GUIDING CENTER ORBITS

Following Littlejohn [13], we begin with the the relativistic phase space Lagrangian for charged particle motion in an electromagnetic field,

$$L(\mathbf{r},\mathbf{u},\dot{\mathbf{r}},\dot{\mathbf{u}},t) = [\frac{e}{m}\mathbf{A}(\mathbf{r},t) + \mathbf{u}]\cdot\dot{\mathbf{r}} - H(\mathbf{u},\mathbf{r},t) \quad , \tag{3}$$

where e is the charge, m is the rest mass, $H = c^2\gamma + e\Phi(\mathbf{r},t)/m$, and $\gamma = (1+u^2/c^2)^{1/2}$. The Euler-Lagrange equations are applied to (3), with $\mathbf{r}$ and $\mathbf{u}$ varied independently. This gives $\dot{\mathbf{r}} = \mathbf{u}/\gamma$, and $m\dot{\mathbf{u}} = e(\mathbf{E} + \mathbf{u}\times\mathbf{B}/\gamma)$, so that $m\mathbf{u}$ is the relativistic momentum. We assume a stationary equilibrium; $\mathbf{A} = \mathbf{A}_0(\mathbf{r})$ and $\Phi = \Phi_0(\mathbf{r})$. The inductive electric field and the RF fields are considered to be perturbations and are treated in the following section.

Let $\mathbf{b}(\mathbf{r})$ denote the unit vector along the equilibrium magnetic field $\mathbf{B}_0(\mathbf{r})$, and let $\mathbf{e}_1(\mathbf{r})$ and $\mathbf{e}_2(\mathbf{r})$ denote an arbitrary pair of perpendicular unit vectors, satisfying $\mathbf{e}_1\times\mathbf{e}_2 = \mathbf{b}$. We decompose $\mathbf{u}$ into parallel and perpendicular components and introduce ϑ as the gyro-angle:

$$\begin{aligned} \mathbf{u} &= u_\parallel \mathbf{b} + u_\perp \mathbf{a}\times\mathbf{b} \\ \mathbf{a} &= \mathbf{e}_1(\mathbf{r})\cos\vartheta - \mathbf{e}_2(\mathbf{r})\sin\vartheta \quad . \end{aligned} \tag{4}$$

Because the scale length of the equilibrium is long compared to the gyroradius, the magnetic moment is an adiabatic invariant. To obtain this invariance, one transforms to a new phase space coordinate system (guiding center coordinates), where the Lagrangian is independent of the gyrophase. The guiding center coordinates are denoted $\mathbf{z} = (\mathbf{x}, U_\parallel, U_\perp, \xi)$ and are related to $(\mathbf{r}, u_\parallel, u_\perp, \vartheta)$ as follows:

$$\begin{aligned} \mathbf{x} &= \mathbf{r} - \epsilon\mathbf{a}\frac{mu_\perp}{eB_0(\mathbf{r})} + \mathcal{O}\epsilon^2 \\ U_\parallel &= u_\parallel + \mathcal{O}\epsilon \\ U_\perp &= u_\perp + \mathcal{O}\epsilon \\ \xi &= \vartheta + \mathcal{O}\epsilon \quad . \end{aligned} \tag{5}$$

Although all quantities have physical units, we have introduced ϵ to signify the ratio of the gyroradius to the scale length of the equilibrium magnetic field, so that it formally indicates the order of various terms in the guiding center expansion (physical equations are obtained by setting ϵ to unity). Higher order terms in the transformation $(\mathbf{r},\mathbf{u}) \to \mathbf{z}$

have been given by Littlejohn [13] and are not needed here. Transforming (3) into guiding center coordinates, one obtains the new Lagrangian [12,13],

$$L(\mathbf{z},\dot{\mathbf{z}},t) = \frac{e}{\epsilon m}\mathbf{A}^* \cdot \dot{\mathbf{x}} + \frac{\epsilon m}{e} M\dot{\xi} - H_0 \quad , \tag{6}$$

where

$$H_0 = c^2\gamma_g + \frac{e}{m}\Phi_0(\mathbf{x}) \tag{7}$$

$$M = \frac{U_\perp^2}{2B_0(\mathbf{x})} \tag{8}$$

$$\mathbf{A}^* = \mathbf{A}_0(\mathbf{x}) + \frac{\epsilon m}{e} U_\parallel \mathbf{b}(\mathbf{x}) \tag{9}$$

and $\gamma_g = [1 + (U_\parallel^2 + U_\perp^2)/c^2]^{1/2}$. The Lagrangian is independent of ξ, so that M (the magnetic moment) is a constant of motion. In stationary fields H_0 is also a constant of motion and it is convenient to take $\mathbf{z} = (\mathbf{x}, M, H_0, \xi)$ as phase space coordinates in (6). In this case $U_\parallel$ is treated as the following function of $\mathbf{z}$:

$$U_\parallel = \sigma\left([H_0 - \frac{e}{m}\Phi_0(\mathbf{x})]^2/c^2 - c^2 - 2MB_0(\mathbf{x})\right)^{1/2} \quad , \tag{10}$$

where $\sigma = \pm 1$. Application of the Euler-Lagrange equations to (6) yeilds the following equations of motion:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$$

$$\dot{\xi} = \frac{eB_0}{\epsilon m \gamma_g} \tag{11}$$

$$\dot{\mathbf{x}} = \frac{U_\parallel \mathbf{B}^*}{\gamma_g \mathbf{B}^* \cdot \mathbf{b}} \quad , \tag{12}$$

where $\mathbf{B}^* = \nabla \times \mathbf{A}^*$. Upon expanding (12) in powers of ϵ and neglecting terms of order ϵ^2, one obtains $\dot{\mathbf{x}} = U_\parallel \mathbf{b}/\gamma_g + \epsilon \mathbf{v}_{g\perp}$, where $\mathbf{v}_{g\perp}$ is the usual expression for the perpendicular (relativistic) guiding center drift [14]. Morozov + Solovie

We assume an axisymmetric equilibrium and express the magnetic field as

$$\mathbf{B}_0 = g(\psi)\nabla\phi + \nabla\psi \times \nabla\phi \quad , \tag{13}$$

where $2\pi\psi$ is the poloidal flux and ϕ is the toroidal angle. Since the toroidal component of $\mathbf{A}_0$ may be written as $\psi\nabla\phi$, one finds [from (6)] the following expression for the toroidal angular momentum:

$$\boxed{p_\phi = \frac{e}{\epsilon m}\psi(\mathbf{x}) + U_\parallel \frac{g(\psi)}{B_0(\mathbf{x})}} \quad . \tag{14}$$

Zero banana width neglects 2nd term in this case

Axisymmetry implies that p_ϕ is also a constant of the unperturbed motion. It is convenient to define a quantity $\bar{\psi}(M, H_0, p_\phi)$ that approximates the instantaneous value of ψ along the guiding center orbit. We define $\bar{\psi}$ implicitly by the relation

$$\bar{\psi} \equiv \frac{\epsilon m}{e}[p_\phi - C(M, H_0, \bar{\psi})] \quad , \tag{15}$$

where $C(M, H_0, \bar{\psi})$ is chosen so that $\bar{\psi}$ has the desired property. Substituting (14) into (15), one finds

$$\bar{\psi} = \psi(\mathbf{x}) + \frac{\epsilon m}{e} \Delta\psi(\mathbf{x}, M, H_0) \tag{16}$$

$$\Delta\psi \equiv U_\| \frac{g(\psi)}{B_0(\mathbf{x})} - C(M, H_0, \bar{\psi}) \quad . \tag{17}$$

E.G. $C = \dfrac{\overline{U_\|(\bar{x})}\,\bar{g}}{\bar{B}_0}$ where $\bar{x}$ is min B pt. on orbit $g \equiv R B_\phi$

We chose $C(M, H_0, \bar{\psi})$ by requiring $\Delta\psi \to 0$ as $B_0(\mathbf{x}) \to B_0(\psi)$, which defines the cylindrical limit. (An appropriate choice for C is given in Appendix A.) As discussed in the introduction, the Fokker-Planck equation is represented in a constant of motion space denoted by the vector $\mathbf{I}$; we now take $I_1 = M$, $I_2 = H_0$, ane $I_3 = \bar{\psi}$.

$\bar{\psi}$ is ψ at min(B)

3. DERIVATION OF THE FOKKER-PLANCK EQUATION

In RF current drive, the electron distribution function $f(\mathbf{u}, \mathbf{r}, t)$ is assumed to obey a kinetic equation of the form

unperturbed

$$(\frac{d}{dt})_0 f + \frac{\partial}{\partial \mathbf{u}} \cdot \mathbf{\Gamma}(f) = 0 \quad , \tag{18}$$

where

$$(\frac{d}{dt})_0 \equiv \frac{\partial}{\partial t} + \frac{\mathbf{u}}{\gamma} \cdot \frac{\partial}{\partial \mathbf{r}} + \frac{e}{m}(\frac{\mathbf{u}}{\gamma} \times \mathbf{B}_0 - \nabla\Phi_0) \cdot \frac{\partial}{\partial \mathbf{u}} \quad . \tag{19}$$

The flux $\mathbf{\Gamma}(f)$ can be decomposed as follows:

$$\mathbf{\Gamma}(f) = \frac{e}{m} E_T \mathbf{e}_\phi f + \mathbf{\Gamma}_{ql}(f) + \mathbf{\Gamma}_c(f) \quad , \tag{20}$$

where E_T is the toroidal electric field induced by the ohmic transformer, $\mathbf{\Gamma}_{ql}$ is the quasilinear flux induced by the RF field, and $\mathbf{\Gamma}_c$ is the collisional flux. The operator $(d/dt)_0$ is the total time derivative along the unperturbed orbits in the equilibrium field. The quasilinear flux has the form

$$\mathbf{\Gamma}_{ql}(f) = \{\frac{e}{m}(\tilde{\mathbf{E}} + \mathbf{u} \times \tilde{\mathbf{B}}/\gamma)\tilde{f}\} \quad , \tag{21}$$

where $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ are the RF fields and $\tilde{f}$ is the fluctuating distribution function, which obeys the linearized Vlasov equation,

$$(\frac{d}{dt})_0 \tilde{f} = -\frac{e}{m}(\tilde{\mathbf{E}} + \mathbf{u} \times \tilde{\mathbf{B}}/\gamma) \cdot \frac{\partial}{\partial \mathbf{u}} f \quad . \tag{22}$$

The curely brackets in (21) denote an ensemble average, or coarse-graining, which annihilates quantities that are linear in the fluctuating fields (it will be more clearly defined in the proceeding section). The collisional flux can be written in the form

$$\mathbf{\Gamma}_c(f) = [\mathbf{F}_c^{\mathbf{u}} - \mathbf{D}_c^{\mathbf{uu}} \cdot \frac{\partial}{\partial \mathbf{u}}] f \quad , \tag{23}$$

where appropriate semi-relativistic forms for the friction vector $\mathbf{F}_c^{\mathbf{u}}$ and the collisional diffusion tensor $\mathbf{D}_c^{\mathbf{uu}}$ have been given by Karney and Fisch [15] and fully relativistic forms have been considered by Braams and Karney [16]. In general, $\mathbf{F}_c^{\mathbf{u}}$ and $\mathbf{D}_c^{\mathbf{uu}}$ also depend on f and the collisional flux is non-linear.

We now consider a phase space transformation from $(\mathbf{u}, \mathbf{r})$ to action-angle coordinates $(\mathbf{J}, \mathbf{\Theta})$. The three actions are denoted as $\mathbf{J}$ and the canonically conjugate angle variables are denoted as $\mathbf{\Theta}$. (The details of this transformation are given in the Appendix B.) In the unperturbed motion, the actions are constant and each canonical angle Θ_i rotates at a constant frequency $\Omega_i(\mathbf{J})$. In particular, Ω_1 is the bounce-averaged gyrofrequency, Ω_2 is the bounce (transit) frequency of a trapped (circulating) particle, and Ω_3 is the bounce-averaged toroidal rotation frequency, which for trapped particles is equivalent to the toroidal drift frequency. Transforming (18) into action-angle coordinates:, one obtains

$$[\frac{\partial}{\partial t} + \Omega_i(\mathbf{J})\frac{\partial}{\partial \Theta_i}]f = -\frac{\partial}{\partial J_i}(\frac{\partial J_i}{\partial \mathbf{u}} \cdot \mathbf{\Gamma}) - \frac{\partial}{\partial \Theta_i}(\frac{\partial \Theta_i}{\partial \mathbf{u}} \cdot \mathbf{\Gamma}) \quad . \tag{24}$$

The left side of (24) is the operator $(d/dt)_0$ represented in action-angle coordintes; the right side is the divergence of $\mathbf{\Gamma}$ in $(\mathbf{J}, \mathbf{\Theta})$ space, since the Jacobian in action-angle coordinates is unity. The angle-average of an arbitrary function, $Y(\mathbf{J}, \mathbf{\Theta}, t)$, is defined as

$$\langle Y \rangle \equiv \frac{1}{(2\pi)^3} \int d^3\Theta \; Y(\mathbf{J}, \mathbf{\Theta}, t) \quad , \tag{25}$$

Noting that $f(\mathbf{J}, \mathbf{\Theta}, t)$ must be periodic in the angles, we angle-average (24) to obtain

$$\frac{\partial}{\partial t} f_0 = -\frac{\partial}{\partial J_i}\langle \frac{\partial J_i}{\partial \mathbf{u}} \cdot \mathbf{\Gamma}(f) \rangle \quad , \tag{26}$$

where $f_0 = \langle f \rangle$. When the difference between f and f_0 is small (owing to the fast rotation of the angles), we may take $f \approx f_0$ in evaluating $\mathbf{\Gamma}(f)$ on the right side of (26), which is thereby reduced to a Fokker-Planck equation in action space alone. This approximation is valid when the bounce (transit) time is much smaller than the characteristic quasilinear-collisional relaxation time. To complete the derivation of (1), we transform (26) from $\mathbf{J}$ space to $\mathbf{I}$ space, where the Jacobian is $\mathcal{J}(\mathbf{I})$ (such that $d^3\mathbf{J} = \mathcal{J} d^3\mathbf{I}$). We thus obtain (1), where the collisional Fokker-Planck coefficients are now given by

$$D_c^{ij} = \langle \frac{\partial I_i}{\partial \mathbf{u}} \cdot \mathbf{D}_c^{\mathbf{uu}} \cdot \frac{\partial I_j}{\partial \mathbf{u}} \rangle \tag{27}$$

$$F_c^i = \langle \frac{\partial I_i}{\partial \mathbf{u}} \cdot \mathbf{F}_c^{\mathbf{u}} \rangle \tag{28}$$

and the ohmic electric field coefficients are

$$F_T^i = \langle \frac{e}{m} E_T \frac{\partial I_i}{\partial \mathbf{u}} \cdot \mathbf{e}_\phi \rangle \quad . \tag{29}$$

The remaining contributions to (1) come from the quasi-linear flux and will be treated in the ~~proceeding~~ following section. From the definitions of **J** and **I**, the Jacobian $\mathcal{J}$ is easily found to be

$$\mathcal{J} = \frac{\tau_b(\mathbf{I})}{2\pi}[1 + \frac{\epsilon m}{e}\frac{\partial C(\mathbf{I})}{\partial I_3}] \quad , \tag{30}$$

where τ_b is the bounce (transit) time for trapped (circulating) orbits. [Note, the Jacobian $\mathcal{J}$ diverges logarythmically (like τ_b) at the trapped-passing boundary. Because the energy is used as a coordinate in **I** space, the inverse transformation $\mathbf{I} \rightarrow \mathbf{J}$ is multi-valued. To resolve this, one must retain the value of σ for circulating orbits.]

To calculate the Fokker-Planck coefficients, we need expressions for $\partial I_i(\mathbf{u}, \mathbf{r})/\partial \mathbf{u}$. Recalling that $I_1 = M$, $I_2 = H_0$, and $I_3 = \bar{\psi}$, one finds the following:

$$\frac{\partial I_1}{\partial \mathbf{u}} = \mathbf{b} \times (\mathbf{u} \times \mathbf{b})/B_0 + \mathcal{O}\epsilon \tag{31}$$

$$\frac{\partial I_2}{\partial \mathbf{u}} = \mathbf{u}/\gamma \tag{32}$$

$$\frac{\partial I_3}{\partial \mathbf{u}} = \frac{\epsilon m}{e}(\frac{\partial \Delta\bar{\psi}}{\partial I_1}\frac{\partial I_1}{\partial \mathbf{u}} + \frac{\partial \Delta\bar{\psi}}{\partial I_2}\frac{\partial I_2}{\partial \mathbf{u}} - \frac{\mathbf{b} \times \nabla\psi}{B_0}) + \mathcal{O}\epsilon^2 \quad . \tag{33}$$

The first two relations above follow directly from the definitions of the magnetic moment and the energy. The third relation follows from the expression for $\bar{\psi}$ given in (16), where we must be careful to calculate $\partial\psi(\mathbf{x})/\partial\mathbf{u}$ at fixed $\mathbf{r}$ (not $\mathbf{x}$) in order to obtain the third term on the right side of (33). The terms involving $\Delta\bar{\psi}$ in (33) give rise to neoclassical transport (recall that $\Delta\bar{\psi}$ vannishes when B_0 is uniform on a flux surface); these terms express the fact that in toroidal geometry, $\bar{\psi}$ shifts in responce to local scattering events in a particle's energy and pitch-angle. The remaining term in (33) gives rise to classical transport and survives in the limit of uniform B_0. In calculating the collisional Fokker-Planck coefficients, one may ignore the classical contribution to (33), since the neoclassical terms are dominant. (Note, this is not necessarily true for the quasilinear contributions, since the Fokker-Planck coefficients depend on the stucture of the RF fields.) Taking note of the definition of $\Delta\bar{\psi}(\mathbf{x}, M, H_0)$ in (17), we find the following relations:

$$\frac{\partial \Delta\bar{\psi}}{\partial I_1} = -\frac{g}{U_\parallel} - \frac{\partial C}{\partial M} \tag{34}$$

$$\frac{\partial \Delta\bar{\psi}}{\partial I_2} = \frac{g\gamma_g}{B_0 U_\parallel} - \frac{\partial C}{\partial H_0} \quad , \tag{35}$$

which determine the neoclassical contributions in (33).

It is convenient to have a representation of the angle average directly in guiding center coordinates (i.e. without reference to the action-angle coordinates). From the definitions of the angles (as given in the Appendix B), it is easily shown that

$$\begin{aligned} \langle Y \rangle &= \tau_b^{-1} \oint dt' \; Y_0[\mathbf{x}(t'), M, H_0, t] \\ Y_0 &\equiv (2\pi)^{-2} \oint d\phi d\xi \; Y(\mathbf{x}, M, H_0, \xi, t) \quad . \end{aligned} \tag{36}$$

where the t' integration follows the unperturbed guiding center orbit for one complete bounce (transit) cycle. [Note, any explicit time dependence of Y_0 is held fixed in the t' integration of (36).]. Operating on axisymmetric, gyrophase invariant functions, the angle average is equivalent to a bounce-average. In practice, the following simplifications may be made in calculating the bounce-average. Consider $\mathbf{x}$ to be a function of the coordinates (ψ, θ, ϕ), where θ is the poloidal angle. Ignoring the perpendicular drift of the guiding center, the t' integration in (36) is converted into θ integration, according to the relation $d\theta/dt' = (U_{\parallel}/\gamma_g)\mathbf{b} \cdot \nabla\theta$. The bounce-time is thus

Zero bana width appro

$$\tau_b \approx \oint \frac{d\theta \, \gamma_g}{U_{\parallel}\mathbf{b} \cdot \nabla\theta} \quad , \tag{37}$$

where the usual care must be taken in handeling the limits of the θ integration for trapped orbits. To the same order of accuracy we may take $\psi \approx \bar{\psi}$ in the integrands of both (36) and (37). Similarly, we may neglect the order ϵ term in (30), so that $\mathcal{J} \approx \tau_b/2\pi$.

4. THE QUASILINEAR DIFFUSION TENSOR

The quasilinear flux in (21) depends on the perturbed distribution function $\tilde{f}$, which can be obtained from the linearized Vlasov equation (22) by the usual method of characteristics (note that f is replaced by f_0 in the driving term). One then finds that the quasi-linear contribution to (1) can be expressed in terms of the diffusion tensor

$$D_{ql}^{ij} = \int_0^{\infty} d\tau \, \langle \{\tilde{V}_i(\mathbf{u}, \mathbf{r}, t)\tilde{V}_j[\mathbf{u}_0(\tau), \mathbf{r}_0(\tau), t - \tau]\}\rangle \quad , \tag{38}$$

where

$$\tilde{V}_i \equiv \frac{e}{m}(\tilde{\mathbf{E}} + \mathbf{u} \times \tilde{\mathbf{B}}/\gamma) \cdot \frac{\partial I_i}{\partial \mathbf{u}} \quad , \tag{39}$$

and $\mathbf{u}_0(\tau), \mathbf{r}_0(\tau)$ is the backward going unperturbed orbit satisfying $\mathbf{u}_0(0) = \mathbf{u}$, $\mathbf{r}_0(0) = \mathbf{r}$. Since I_i is conserved along the unperturbed orbit, it follows from (39) that $dI_i/dt = \tilde{V}_i$. Hence, (38) is just the expected relation between the diffusion tensor and the Lagrangian auto-correlation function.

Rather than calculating $\tilde{V}_i$ from (39), we find expressions for $\tilde{V}_i$ directly in guiding center coordinates, which is convenient for performing the orbit integration and the angle-average required by (38). For simplicity, we first assume electrostatic fluctuations and later gerneralize to treat the case of electromagnetic fluctuations. In the electrostatic case, the RF fields are easily included in the guiding center Lagrangian (6) by adding $e\tilde{\Phi}/m$ to the Hamiltonian, where $\tilde{\Phi}$ is the electrostatic potential of the waves. Using the modified Lagrangian and noting the relation $\tilde{V}_i = dI_i/dt$, as well as the definitions $I_1 = M$, $I_2 = H_0$, and $I_3 = \bar{\psi}$, the following equations are obtained:

$$\tilde{V}_1 = -\frac{e}{\epsilon m}\frac{\partial}{\partial \xi}\tilde{H}(\mathbf{z}, t) \tag{40}$$

$$\tilde{V}_2 = \frac{\partial}{\partial t}\tilde{H}(\mathbf{z}, t) - (\frac{d}{dt})_0\tilde{H} \tag{41}$$

$$\tilde{V}_3 = \frac{\epsilon m}{e}[\frac{\partial \Delta\bar{\psi}}{\partial I_1}\tilde{V}_1 + \frac{\partial \Delta\bar{\psi}}{\partial I_2}\tilde{V}_2 - (\frac{\mathbf{b} \times \nabla\psi}{B_0}) \cdot \frac{\partial}{\partial \mathbf{x}}\tilde{H}(\mathbf{z}, t)] \quad , \tag{42}$$

where $\tilde{H}(\mathbf{z},t) = e\tilde{\Phi}[\mathbf{r}(\mathbf{z}),t]/m$. In writing (41), we have neglected terms that are nonlinear in the wave amplitude and in obtaining (42), we have neglected terms of order ϵ^2. The terms involving $\Delta\bar{\psi}$ in (42) are neoclassical; the remaining term is the local (radial) guiding center drift induced by the fluctuating fields and it survives in the cylindrical limit.

To treat the fully electromagnetic case, we absorb the wave vector potential $\tilde{\mathbf{A}}$ into the perturbed Hamiltonian by redefining phase space coordinates,

$$\mathbf{u}' = \mathbf{u} + \frac{e}{m}\tilde{\mathbf{A}}(\mathbf{r},t) \quad , \tag{43}$$

so that the total Hamiltonian [see (3)] can be written as $H = c^2\gamma' + e\Phi_0(\mathbf{r})/m + \tilde{H}$, where $\gamma' = [1 + (u'/c)^2]^{1/2}$ and

$$\begin{aligned} \tilde{H} &= \frac{e}{m}\tilde{\Phi}(\mathbf{r},t) + c^2(\gamma - \gamma') \\ &\approx \frac{e}{m}[\tilde{\Phi}(\mathbf{r},t) - \tilde{\mathbf{A}}(\mathbf{r},t)\cdot\mathbf{u}'/\gamma'] \quad . \end{aligned} \tag{44}$$

The second expression for $\tilde{H}$ is obtained by linearizing with respect to the wave amplitude. Following Littlejohn [13], the guiding center transformation is now implemented with respect to $\mathbf{u}'$ instead of $\mathbf{u}$, so that the guiding center Lagrangian takes on the same form as in (6), but including $\tilde{H}$ as the perturbed Hamiltonian. Since the definition of the coordinates now include the perturbed fields, one must be careful to distinguish between $I_i(\mathbf{u},\mathbf{r})$ and the related quantity $I_i' \equiv I_i(\mathbf{u}',\mathbf{r})$. Defining $\tilde{V}_i' = dI_i'/dt$, one finds that $\tilde{V}_i'$ is related to $\tilde{H}$, by equations which are of the same form as (40)-(42). The diffusion tensor, however, has been defined with respect to $\tilde{V}_i$. To first order in the wave amplitude, one finds that

$$\tilde{V}_i = \tilde{V}_i' - \frac{e}{m}(\frac{d}{dt})_0\tilde{\mathbf{A}}\cdot\frac{\partial I_i}{\partial \mathbf{u}} \quad . \tag{45}$$

The second term in the above expression does not contribute to the quasilinear diffusion tensor, because the τ integration in (38) forces all contributions to be evaluated in the resonant limit, where $(d/dt)_0 \to 0$. Thus, when calculating D_{ql}^{ij} we may use (40)-(42) directly, where $\tilde{H}$ is given by (44). [Notice, that the same argument concerning the resonance condition allows one to ignore the term $(d/dt)_0\tilde{H}$ in (41).]

In RF heating and current drive, the waves are driven by an external source at frequency ω, so we write the RF electric field as

$$\tilde{\mathbf{E}}(\mathbf{r},t) = \tilde{\mathbf{E}}(\mathbf{r})e^{-i\omega t} \quad + \text{c.c.} \quad . \tag{46}$$

The perturbed Hamiltonian in (44) can be written in terms of the wave electric field by chosing the gauge condition $\tilde{\Phi} = 0$. Expressing $\tilde{H}$ in guiding center coordinates and ignoring terms of order ϵ, one finds

$$\tilde{H} = -\frac{e}{m\gamma_g}(U_\parallel\mathbf{b} + U_\perp\mathbf{a}\times\mathbf{b})\cdot\tilde{\mathbf{E}}(\mathbf{x}+\rho\mathbf{a})\frac{e^{-i\omega t}}{i\omega} \quad + \text{c.c.} \quad , \tag{47}$$

where $\rho \equiv mU_\perp/eB_0(\mathbf{x})$ and the unit vectors are now evaluated at the guiding center. Since the perpendicular scale length of the waves can be much shorter than the scale length of the equilibrium field, we retain the finite gyroradius term in evaluating the wave field. We stress that in calculating the partial derivatives in (40)-(42), the independent variables of $\tilde{H}$ are $(\mathbf{x}, M, H_0, \xi, t)$.

We now consider the quasilinear diffusion tensor for cases when the wave field can be described by geometric optics. We thus assume an eikonal form,

$$\tilde{\mathbf{E}}(\mathbf{r}) = \sum_s \mathbf{E}_s(\mathbf{r}) \exp iS_s(\mathbf{r}) \quad , \tag{48}$$

where $\mathbf{E}_s(\mathbf{r})$ and $\mathbf{k}_s(\mathbf{r})$ are assumed to be slowly varying (note, $\mathbf{k}_s \equiv \nabla S_s$), and the subscript s refers to the s^{th} ray in the wave packet. In evaluating $\tilde{\mathbf{E}}(\mathbf{x}+\rho\mathbf{a})$, we ignore terms of order $\rho/\mathcal{L}$, where $\mathcal{L}$ is the lengthscale of the slow variations (we take $\rho/\mathcal{L}$ to be of order ϵ). Supressing the subscript s, we decompose the wavevector into parallel and perpendicular components and we define the angle φ by the relation $\mathbf{a}\cdot\mathbf{k} = k_\perp \sin(\xi+\varphi)$. Starting from (47), it is now straight forward to express the perturbed Hamiltonian in the following form (as given by Littlejohn [13]):

$$\tilde{H} = \sum_s \sum_{n=-\infty}^{\infty} H_{sn}(\mathbf{x}, M, H_0) \exp i[S_s(\mathbf{x}) + n(\xi+\varphi_s) - \omega t] \quad + \text{c.c.} \quad , \tag{49}$$

where

$$H_{sn} = -\frac{e}{m}\frac{\mathbf{E}_s(\mathbf{x})}{i\omega\gamma_g} \cdot [U_\parallel J_n \mathbf{b} + U_\perp(\frac{nJ_n}{k_\perp \rho}\hat{\mathbf{k}}_\perp + iJ_n' \mathbf{b}\times\hat{\mathbf{k}}_\perp)] \quad . \tag{50}$$

Here J_n is the order n Bessel function with argument $k_\perp\rho$ and J_n' is its derivative with respect to argument. Also $\hat{\mathbf{k}}_\perp \equiv \mathbf{b}\times(\mathbf{k}\times\mathbf{b})/k_\perp$.

The quasilinear diffusion tensor is calculated according to (38), where the functions $\tilde{V}_i$ are expressed in (40)-(42) and the perturbed Hamiltonian is expressed in (49). The exact result is quite complicated, because both the slow and fast variations are included in the τ integration along the unperturbed orbit. The situation simplifies if $\mathbf{x}_0(\tau)$ can be replaced by $\mathbf{x}$ in all slowly varying quantities. Assuming the integrand in (38) falls off rapidly for $\tau > \tau_{ac}$, we may make the above replacement when $|\mathbf{x}_0(\tau_{ac}) - \mathbf{x}| \ll \mathcal{L}$. Since the integrand in (38) is quadratic in $\tilde{H}$, it contains terms of the form $H_{sn}H^*_{s'n'}$ (and $H_{sn}H_{s'n'}$), which are then summed over all indices n and n', aswell as over all rays s and s'. These terms are subject to both an ensemble-averaging and an angle-averaging, where the later is carried out according to the prescription given in (36). The ensemble average is now defined as a coarse-graining in time and space (at the time scale of a wave period and the length scale of several wavelengths). The time average removes terms which oscillate at 2ω and the coarse-graining in space tends to reduce the double sum over rays to terms with $s = s'$. The angle-averaging includes an average over the gyrophase, which reduces n' to n. The remaining slow variations are then averaged over the bounce-orbit according to (36). Making the above simplifications results in the following expression for the quasilinear diffusion tensor:

$$D_{ql}^{ij}(\mathbf{I}) = \langle \mathcal{D}_{ql}^{ij}(\mathbf{x}, M, H_0) \rangle \quad , \tag{51}$$

$$\mathcal{D}_{ql}^{ij} = \sum_s \sum_n \alpha_{sn}^i \alpha_{sn}^j |H_{sn}|^2 R_{sn} \quad , \tag{52}$$

where the vector α_{sn}^i has three components, $\alpha_{sn}^1 \equiv n(e/\epsilon m)$, $\alpha_{sn}^2 \equiv \omega$, and

$$\alpha_{sn}^3 \equiv \frac{\epsilon m}{e}[\frac{\partial \Delta \bar{\psi}}{\partial I_1}\alpha_{sn}^1 + \frac{\partial \Delta \bar{\psi}}{\partial I_2}\alpha_{sn}^2 + (\frac{\mathbf{b} \times \nabla \psi}{B_0}) \cdot \mathbf{k}_s] \quad . \tag{53}$$

In (52) we have also introduced R_{sn} as the resonance function,

$$R_{sn} \equiv \int_0^\infty d\tau \ \exp i[\omega\tau - \mathbf{k}_s \cdot \Delta\mathbf{x}(\tau) - n\Delta\xi(\tau)] \quad + \text{c.c.} \quad , \tag{54}$$

where $\Delta\mathbf{x}(\tau) = \mathbf{x} - \mathbf{x}_0(\tau)$ and $\Delta\xi(\tau) = \xi - \xi_0(\tau)$. Assuming that $\tau_{ac} \ll \tau_b$, we may take $\Delta\mathbf{x}(\tau) \approx \tau\mathbf{v}_g$ and $\Delta\xi(\tau) \approx \tau\Omega_{ce}$, where $\mathbf{v}_g = U_\parallel \mathbf{b}/\gamma_g + \epsilon\mathbf{v}_{g\perp}$ and $\Omega_{ce} = eB_0/m\gamma_g$. Substituting these approximations into (54), one obtains the local resonance condition

$$R_{sn} = 2\pi\delta(\omega - \mathbf{k}_s \cdot \mathbf{v}_g - n\Omega_{ce}) \quad . \tag{55}$$

Note, in the usual RF heating and current drive schemes, the perpendicular drift velocity makes a negligible modification to the local resonance condition. The validity of the above analysis can be confirmed by calculating

$$\tau_{ac} \sim \langle \sum_{s,n} |H_{sn}|^2 R_{sn} \rangle / \langle \sum_{s,n} |H_{sn}|^2 \rangle \tag{56}$$

to check that $\tau_{ac} \ll \tau_b$. It is worthwhile pointing out that in the case of cyclotron heating, R_{sn} must be modified for trapped particles whose banana tips are in the cyclotron resonance layer [17].

The quasilinear diffusion tensor in (51) appears as the bounce-average of a local diffusion tensor, $\mathcal{D}_{ql}^{ij}(\mathbf{x}, M, H_0)$, which can be evaluated with the aid of conventional ray tracing techniques, as carried out in Ref.[10] for LHCD. The local diffusion tensor includes both classical and neoclassical drifts, as given by the expression for α_{sn}^3 in (53). Examining this expression, one finds that the classical drifts can dominate when $\mathbf{k}_s$ develops a large poloidal component, as can occur because of toroidal effects on the ray propagation during multipass absorption. Since α_{sn}^3 is of order ϵ relative to α_{sn}^1 and α_{sn}^2, the diffusion takes place mostly in the (M, H_0) plane, at fixed $\bar{\psi}$. In the limit $\alpha_{sn}^3 \to 0$, the local diffusion tensor given by (52) is equivalent to the usual expression for velocity space diffusion in a locally uniform magnetic field (see Appendix C).

We note that an alternative proceedure for calculating the quasilinear diffusion tensor is to use the action-angle representation in (38). Upon making the replacement $\tilde{V}_i \to -\partial\tilde{H}(\mathbf{J}, \Theta, t)/\partial\Theta_i$ in (38), one obtains the conventional expression for the diffusion tensor in action space [18], which takes on an especially simple form if we write $\tilde{H}$ as a Fourier series with respect to the angle coordinates. In this case, the complexity is hidden in the calculation of the Fourier coefficients.

ACKNOWLEDGEMENTS

I would like to acknowledge Prof. Abraham Bers for many helpful discussions during the development of this work. Also, I would like to acknowledge Dr. Robert Harvey for discussions concerning the possibility of modifying the CQL3D Fokker-Planck code to include the transport effects discussed in this paper.

APPENDIX A

Here we discuss an appropriate choice for the quantity $C(M, H_0, \bar{\psi})$, which determines our definition of $\bar{\psi}$. Consider the unperturbed guiding center orbit $\mathbf{x}(t)$ as it passes through the point $\bar{\mathbf{x}}$, where the equilibrium magnetic field takes on its minimum value; we define $\bar{\psi} = \psi(\bar{\mathbf{x}})$. Thus $\bar{\psi}$ is the poloidal flux function evaluated at the outer most point along the bounce orbit. From (16) and (17), we find that the above definition of ψ corresponds to the following choice for $C(M, H_0, \bar{\psi})$:

$$C(M, H_0, \bar{\psi}) = \bar{U}_{\parallel} \frac{g(\bar{\psi})}{\bar{B}_0(\bar{\psi})} \quad , \tag{A.1}$$

where $\bar{B}_0(\psi)$ is the minimum value of B_0 on any flux surface ψ and

$$\bar{U}_{\parallel} = \bar{\sigma}[H_0^2/c^2 - c^2 - 2M\bar{B}_0(\bar{\psi})]^{1/2} \quad . \tag{A.2}$$

(For simplicity, we have assumed $\Phi_0 = 0$ in the above relation for $\bar{U}_{\parallel}$.) Here $\bar{\sigma}$ is the value of σ at $\mathbf{x}(t) = \bar{\mathbf{x}}$. For circulating electrons $\bar{\sigma} = \sigma = \pm 1$. All trapped electrons share the same value of $\bar{\sigma}$, with inner and outer banana orbits being distinguished by small differences in the value of $\bar{\psi}$. It is worthwhile pointing out that a reasonable alternative definition for ψ is the bounce-averaged flux surface, $\langle \psi(\mathbf{x}) \rangle$, which corresponds to the choice $C = \langle U_{\parallel} g(\psi)/B_0(\mathbf{x}) \rangle$. Using this definition, however, may cause numerical problems in the vicinity of the trapped-passing boundary due to the divergence of $\partial C/\partial M$ and $\partial C/\partial H_0$, which appear in (34) and (35).

APPENDIX B

Here we consider the construction of action-angle coordinates $(\mathbf{J}, \mathbf{\Theta})$. The action-angle coordinates are obtained by a sequence of transformations, $(\mathbf{u}, \mathbf{r}) \rightarrow (\mathbf{z}) \rightarrow (\mathbf{I}, \mathbf{\Theta})$, where the guiding center coordinates $(\mathbf{z})$ are introduced as an intermediate step, to remove the gyrophase dependence in the Lagrangian, according to the discussion of Section 2. To put the guiding center Lagrangian into canonical form, one constructs a set of toroidal coordinates, which allow the following representation of the magnetic field:

$$\mathbf{B}_0 = \mathcal{B}_\phi \nabla\zeta_\phi + \mathcal{B}_\theta \nabla\zeta_\theta \tag{B.1}$$

$$\mathbf{A}_0 = \mathcal{A}_\phi \nabla\zeta_\phi + \mathcal{A}_\theta \nabla\zeta_\theta \tag{B.2}$$

where ζ_θ and ζ_ϕ are generalized poloidal and toroidal angles. In the $(\psi, \zeta_\theta, \zeta_\phi)$ coordinates, the magnetic field has no covariant component in the $\nabla\psi$ direction, as is evident in (B.1). A general procedure for constructing toroidal coordinates with this property has been given by Meiss and Hazeltine [19] and the details are not needed here. Substituting (B.1) and (B.2) into (6), one finds that

$$L = p_\phi \dot{\zeta}_\phi + p_\theta \dot{\zeta}_\theta + p_\xi \dot{\xi} - H_0 \quad , \tag{B.3}$$

where $p_\xi = \epsilon m M/e$ and the momenta conjugate to ζ_ϕ and ζ_θ are

$$p_\phi = \frac{e}{\epsilon m} \mathcal{A}_\phi(\mathbf{x}) + U_\| \frac{\mathcal{B}_\phi(\mathbf{x})}{B_0(\mathbf{x})} \tag{B.4}$$

$$p_\theta = \frac{e}{\epsilon m} \mathcal{A}_\theta(\mathbf{x}) + U_\| \frac{\mathcal{B}_\theta(\mathbf{x})}{B_0(\mathbf{x})} \tag{B.5}$$

Noting that $\mathcal{A}_\phi = \psi$ and $\mathcal{B}_\phi = g(\psi)$, one recovers the expression for p_ϕ in (14).

The actions are defined in terms of the canonical momenta. We adopt the convention $J_1 = p_\xi$ and $J_3 = p_\phi$. The action J_2 is defined so that $2\pi J_2$ is the area in the (p_θ, ζ_θ) plane enclosed by a curve of constant H_0, p_ϕ, and p_ξ. (Note, for circulating orbits it is the area beneath this curve.) Thus I_2 is written as

$$I_2 = (2\pi)^{-1} \oint d\zeta_\theta \; p_\theta(\zeta_\theta; H_0, p_\phi, p_\xi) \quad , \tag{B.6}$$

where the integration in ζ_θ is over one complete bounce cycle. Through (B.6), the unperturbed Hamiltonian can be expressed in the form $H_0(\mathbf{J})$. The canonical angles are obtained from the mixed variable generating function [20],

$$G(\zeta_\theta, \zeta_\phi, \xi, \mathbf{J}) = \int^{\zeta_\theta} d\zeta_\theta \; p_\theta(\zeta_\theta, \mathbf{J}) \; + J_1 \xi \; + J_3 \zeta_\phi \quad , \tag{B.7}$$

by differentiating with respect to the actions,

$$\Theta_i = \frac{\partial G}{\partial J_i} \quad . \tag{B.8}$$

The unperturbed frequencies are

$$\dot{\Theta}_i = \frac{\partial H_0}{\partial J_i} \equiv \Omega_i(\mathbf{J}) \quad . \tag{B.9}$$

The difference between the above treatment and the one originally given by Kaufman [18], is contained in our expression for p_θ. We see from (B.5) that p_θ includes a term proportional to $\mathcal{B}_\theta$, which properly accounts for the effect of the poloidal magnetic field on the guiding center drift.

APPENDIX C

We now show that $\mathcal{D}_{ql}^{ij}$ in (52) corresponds to the usual quasilinear velocity spac diffusion in a locally uniform magnetic field. First note that $\mathcal{D}_{ql}^{ij}$ depends on $|H_{sn}|^2$, wher H_{sn} is defined in (50). Defining the complex unit vectors

$$\mathbf{e}_+ = (\hat{\mathbf{k}}_\perp - i\mathbf{b} \times \hat{\mathbf{k}}_\perp)/\sqrt{2} \tag{C.1}$$

$$\mathbf{e}_- = (\hat{\mathbf{k}}_\perp + i\mathbf{b} \times \hat{\mathbf{k}}_\perp)/\sqrt{2} \tag{C.2}$$

one finds that

$$H_{sn} = -\frac{e}{m}\frac{\mathbf{E}_s}{i\omega\gamma} \cdot [u_\parallel J_n \mathbf{b} + u_\perp(\mathbf{e}_+ J_{n+1} + \mathbf{e}_- J_{n-1})/\sqrt{2}] \quad . \tag{C.3}$$

where we have let $U_\parallel \rightarrow u_\parallel$, $U_\perp \rightarrow u_\perp$ and $\gamma_g \rightarrow \gamma$, as is appropriate in the limit of uniforr magnetic field. If one now lets $I_1 \rightarrow u_\perp^2/2B_0$ and $I_2 \rightarrow c^2\gamma$, one finds that the quasilinea diffusion equation in $(u_\perp, u_\parallel)$ space has the following form:

$$\frac{\partial}{\partial t} f_0(u_\perp, u_\parallel, t) = \sum_s \sum_n G_{sn} D_{sn} L_{sn} f_0(u_\perp, u_\parallel, t) \quad , \tag{C.4}$$

where

$$D_{sn} = 2\pi\delta(\omega - u_\parallel k_\parallel - n\Omega_{ce})\frac{\omega^2\gamma^2}{u_\perp^2}|H_{sn}|^2 \tag{C.5}$$

and we have defined the operators

$$G_{sn} = \frac{1}{u_\perp}\frac{\partial}{\partial u_\perp} u_\perp(\frac{n\Omega_{ce}}{\omega}) + \frac{\partial}{\partial u_\parallel}(\frac{k_\parallel u_\perp}{\omega\gamma}) \tag{C.6}$$

$$L_{sn} = (\frac{n\Omega_{ce}}{\omega})\frac{\partial}{\partial u_\perp} + (\frac{k_\parallel u_\perp}{\omega\gamma})\frac{\partial}{\partial u_\parallel} \quad . \tag{C.7}$$

Note, that in obtaining (C.4) we have ignored the perpendicular guiding center drift b letting $\alpha_{sn}^3 \rightarrow 0$ in (52). Equation (C.4) is the relativistic generlization of the velocit space quasilinear diffusion equation obtained by Kennel and Engelmann [21]. The noı relativistic limit is taken by letting $\gamma \rightarrow 1$ and replacing $u_\perp$ and $u_\parallel$ with the ordinar perpendicular and parallel velocities.

REFERENCES

[1] J.M. Rax, D. Moreau, Nucl. Fusion **29**, 1751 (1989).
[2] M.R. O'Brien, M. Cox, and J.S. McKenzie, Nucl. Fusion **31**, 583 (1991).
[3] P.T. Bonoli, M. Porkolab, Y. Takase, and S.F. Knowlton, Nucl. Fusion **28** 991 (1988).
[4] V. Fuchs, I.P. Shkarofsky, R.A. Cairns, and P.T. Bonoli, Nucl. Fusion **29** 1479 (1989).
[5] R.W. Harvey, M.G. McCoy, G.D. Kerbel, and G.R. Smith, "The CQL3D Fokker-Planck Model for Tokamaks", International Sherwood Fusion Conference, April 22-24, 1991, Seattle, Washington.
[6] G. Giruzzi, "Fokker-Planck Calculations of Fast Wave Current Drive", IAEA Technical Committee Meeting on Fast Wave Current Drive in Reactor Scale Tokamaks, September 23-25, 1991, Arles, France.
[7] M. Cox, "3D Calculations of LHCD/FWCD Synergism Including Radial Transport Effects", IAEA Technical Committee Meeting on Fast Wave Current Drive in Reactor Scale Tokamaks, September 23-25, 1991, Arles, France.
[8] T.M. Antonsen, Jr., and K. Yoshioka, Phys. Fluids **29**, 2235 (1986).
[9] X. Meng-Fen and W. Wei-Min, Plasma. Phys. and Contr. Fusion **29**, 621 (1987).
[10] K. Kupfer and A. Bers, "Fast Electron Transport in Lower-Hybrid Current Drive", to be published in Phys. Fluids B, October 1991.
[11] I.B. Bernstein and K. Molvig, Phys. Fluids **26**, 1488 (1983).
[12] R.G. Littlejohn, J. Plasma Phys. **29**, 111 (1983).
[13] R.G. Littlejohn, Phys. Fluids **27**, 976 (1984).
[14] A.I. Morozov and L.S. Soloviev, in *Review of Plasma Physics*, Vol. 2, edited by A. Leontovich, Consultants Bureau, New York (1965).
[15] C.F.F. Karney and N.J. Fisch, Phys. Fluids **28**, 116 (1985).
[16] B.J. Braams and C.F.F Karney, "Differential Form of the Collision Integral for a Relativistic Plasma", Princeton Plasma Physics Laboratory, Report PPPL-2467, August 1987, Princeton, New Jersey.
[17] G.D. Kerbel and M.G. McCoy, Phys. Fluids **28**, 3629 (1985).
[18] A.N. Kaufman, Phys. Fluids **15**, 1063 (1972).
[19] J.D. Meiss and R.D. Hazeltine, Phys. Fluids B **2**, 2563 (1990).
[20] H. Goldstein, *Classical Mechanics*, 2nd ed., Addison-Wesley (1980).
[21] C.F. Kennel and F. Engelmann, Phys. Fluids **9**, 2377 (1966).