APS, Minneapolis, 1994.

7R35 Fokker-Planck Simulations of Parallel Electron Transport in the Scrape-Off Layer,* K. Kupfer,† R.W. Harvey, O. Sauter,‡ G.M. Staebler, *General Atomics*—Electron transport in the diverted scrape-off layer of a tokamak is studied using a 3D Fokker-Planck code (FPET) with two dimensions in velocity space and one spatial dimension along the magnetic field line. This provides a fully kinetic treatment of the electron distribution function, necessary because the mean free path of energetic electrons is long with respect to the length of the field line. The electrostatic potential is calculated self-consistently, including a jump condition at the plasma-sheath interface. The ions are taken to be a fixed fluid background. In the long mean free path regime, the parallel electron heat flux is often prescribed by a simple local formula combining the classical conductive heat flux and a free-streaming flux limit [1]. A comparison is made between our Fokker-Planck results and various local and non-local fluid models for the parallel electron heat flux, including the flux-limiting condition.

*Research, work supported by: the U.S. DOE under Grant DE-FG03-92ER54150, Contract DE-AC03-89ER51115; in part by appointment to the U.S. DOE Fusion Energy Postdoctoral Research Program administered by ORISE; and the Swiss National Science Foundation; such financial support does not constitute an endorsement by DOE of the views expressed herein.

†ORISE Postdoctoral Fellow. ‡CRPP/EPFL, Lausanne.

[1] S.A. Kahn, T.D. Rognlien, Phys. Fluids *24* (1981) 1442.

FOKKER-PLANCK EDGE TRANSPORT CODE (FPET)

- Electrons in the diverted scrape-off layer of a tokamak are typically in a mixed collisionality regime
  - — bulk electrons are moderately collisional,
  - — tail electrons are nearly collisionless.

- Kinetic effects play an important role in the parallel transport of heat and in determining the electric field, which effects the ion flow rate and impact energy on the divertor plate.

- Non-Maxwellian distortions of the electron distribution function effect the power radiated by impurities.

- FPET is a fully kinetic parallel transport code which solves for $f(v_\perp, v_\parallel, z)$, the gyro-averaged distribution function along a magnetic field line. Presently, we solve for the electron distribution function assuming a fixed ion background.

- FPET calculates the **self-consistent electric field**.

RESULTS

- FPET recovers classical transport results in the short mean free path regime.

- For large temperature gradients, FPET calculations of the electron heat flux agree with previously published results [2]. The departure from classical transport theory (due to finite mean free path effects) appears in **both** the heat flux **and the electric field**.

- For typical SOL conditions in a DIII-D H-mode plasma, we have used FPET to calculate (i) the electron heat flux, (ii) the self-consistent electric field in the pre-sheath, and (iii) the self-consistent potential drop across the sheath. The calculated heat flux agrees reasonably well with previous Monte Carlo simulations [3].

CONCLUSION

Existing fluid models do not properly describe parallel transport in the SOL under semi-collisionless conditions. **This includes the flux limited model.**

[2] J.F. Luciani, P. Mora, J. Virmont, Phys. Rev. Lett. **51**, 1664 (1983).

[3] R.H. Cohen, T.D. Rognlien, "Finite Mean-Free-Path Effects in Tokamak Scrape-Off Layers," Contr. Plasma Phys. (1993).

FUTURE PLANS

Main objective —

(1) Identify areas where kinetic effects play an important role in edge physics.

(2) Develop appropriate methods for including these kinetic effects in existing edge modeling codes.

Example — Develop a multi-parameter fit for the particle flux and heat flux as functions of the density, temperature, and electric field (including the sheath potential).

Upgrade the collision operator —

FPET presently implements a test particle collision operator. The linearized momentum conserving operator and the full non-linear collision operator are being extracted from the CQL3D code.

Develop a reduced 2D fluid/kinetic model —

One attempts to replace the perpendicular velocity dependence in the 3D kinetic equation by an appropriate moment expansion, producing a hybrid 2D fluid/kinetic approach.

Preliminary results have been achieved by taking the distribution function to be Maxwellian (at the background temperature) with respect to perpendicular velocity.

Couple ions and electrons —

FPET presently assumes a fixed ion background. We plan to treat the ion dynamics in a self-consistent fashion, either as a fluid, or as a second kinetic species.

Include cross-field transport —

The simplest approach is to treat cross-field transport effects as a distributed source/sink of particles along a single field line.

A more complete approach is to add cross-field transport terms directly to the kinetic equation, extending the configuration space from one to two dimensions. The required computer time scales roughly linearly with the total number of grid points in configuration space.

This problem is ideal for parallel processing, since most of the CPU time is spent inverting a 2D velocity space operator, which can be done simultaneously at all spatial grid points.

A more practical approach may be to incorporate cross-field transport as an extension of the 2D fluid/kinetic model (discussed above).

3D Kinetic Equation

$$\left(\frac{\partial}{\partial t} + v_{\|}\frac{\partial}{\partial z} + E_{\|}\frac{\partial}{\partial v_{\|}}\right) f_e = C(f_e) \; + \; \text{sources}$$

Collision Operator

(1) Test particle operator.
(2) Linearized momentum conserving operator.
(3) Full non-linear integral operator.

Electric Field

(1) Poisson's equation

$$(\frac{\partial}{\partial z})E_{\|} = \frac{e}{\epsilon_o}\left(n_i - Z_i \int d^3v \; f_e\right)$$

(2) Quasi-neutrality (flux constraint)

$$\int d^3v \; v_{\|} \; f_e = \text{constant}$$

Numerical Method

Define configuration space and velocity space operators:

$$A(f) = (v_{\|}\frac{\partial}{\partial z})f$$

$$B(E_{\|}, f) = (E_{\|}\frac{\partial}{\partial v_{\|}})f - C(f)$$

Use ADI method to time step the kinetic equation:

Step 1

$$(f^{n+1/2} - f^n)2/\Delta t \; + \; A(f^{n+1/2}) = -B(E_{\|}, f^n)$$

Step 2

$$(f^{n+1} - f^{n+1/2})2/\Delta t \; + \; B(E_{\|}, f^{n+1}) = -A(f^{n+1/2})$$

- Most of the CPU time is spent inverting the 2D velocity space operator in Step 2, which involves solving a large sparse system of linear equations.

- Steady-state solutions are typically obtained in 30 to 60 time-steps, totaling about 10 thermal collision times.

- Run-times on the HP 9000 T500 workstation, using a 40x50 velocity space mesh, are typically 30 minutes to 1 hour.

Solving Poisson's Equation

The **steady-state** solution of Poisson's equation is obtained by relaxation. The distribution function is updated from f^n to f^{n+1} using the **time retarded** electric field, $E_{\|}^n$.

The electric field is advanced as follows

$$\gamma(\phi^{n+1} - \phi^n) - (\frac{\partial^2}{\partial z^2})\phi^{n+1} = \frac{e}{\epsilon_o}\left(n_i - Z_i \int d^3v \; f^n\right)$$

where $E_{\|}^n = -\partial\phi^n/\partial z$ and γ is a numerical parameter that determines the relaxation rate.

- The potential and the distribution function gradually adjust to one another and to the boundary conditions.
- We have used this algorithm with absorbing wall boundary conditions to obtain sheath solutions for the regime $\lambda_{MFP}/\lambda_D \lesssim 10$. (Here λ_{MFP} is the mean free path of thermal electrons and λ_D is the Debye length.) To avoid numerical instabilities, we typically had to set $\gamma \sim (\lambda_{MFP}/\lambda_D)^2$.
- Although the stability boundaries of this algorithm need to be quantified more carefully, it provides one possible method for obtaining kinetic solutions in the sheath.

Imposing Quasi-Neutrality

In the pre-sheath, the self-consistent **slowly varying** electric field may be calculated by imposing quasi-neutrality. The distribution function is updated from f^n to $f^{n+1/2}$ using the **time advanced** electric field, $E_{||}^{n+1}$.

To determine $E_{||}^{n+1}$, we define the reduced (2D) distribution function

$$F(v_{||}, z) = 2\pi \int v_\perp dv_\perp \, f$$

Setting $E_{||}^{n+1} = E_{||}^{n} + \Delta E_{||}$ in the first ADI step and integrating over perpendicular velocity yields

$$(F^{n+1/2} - F^n)2/\Delta t + A(F^{n+1/2}) + \Delta E_{||}\left(\frac{\partial}{\partial v_{||}}\right)F^n = - 2\pi \int v_\perp dv_\perp \, B(E_{||}^{n}, f^n) \quad .$$

Simultaneous solutions for both $F^{n+1/2}$ and $\Delta E_{||}$ are determined by imposing a **constraint on the flux**

$$\int dv_{||} \, v_{||} F^{n+1/2} = \text{constant} \quad .$$

Once $E_{||}^{n+1}$ is determined, FPET calculates $f^{n+1/2}$ and f^{n+1} by the prescribed two-step ADI method. The flux constraint is not imposed on the second step, i.e., when calculating f^{n+1} we do not re-calculate the electric field.

SOL Example Calculation

- **Parameters** — DIII-D H-mode plasma.

- **Boundary Conditions** —

  Mid-plane: we impose a symmetry condition.

  Wall: electrons with $(m_e v_{\|}^2/2) < e\Delta\phi_{\rm sh}$ are reflected, otherwise they are absorbed.

- **Source** —

  At the mid-plane, a half-Maxwellian distribution of electrons is injected (moving towards the wall).

- **Sheath Potential** —

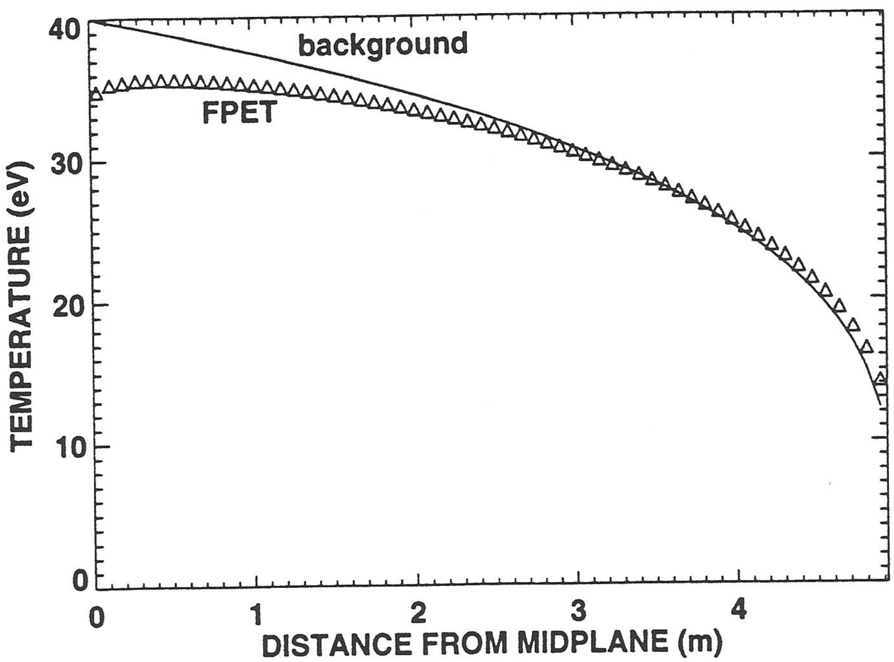
  The ions are assumed to have a flow velocity of Mach 1 at the sheath edge. The sheath potential drop $\Delta\phi_{\rm sh}$ is calculated by balancing electron and ion currents to the wall.

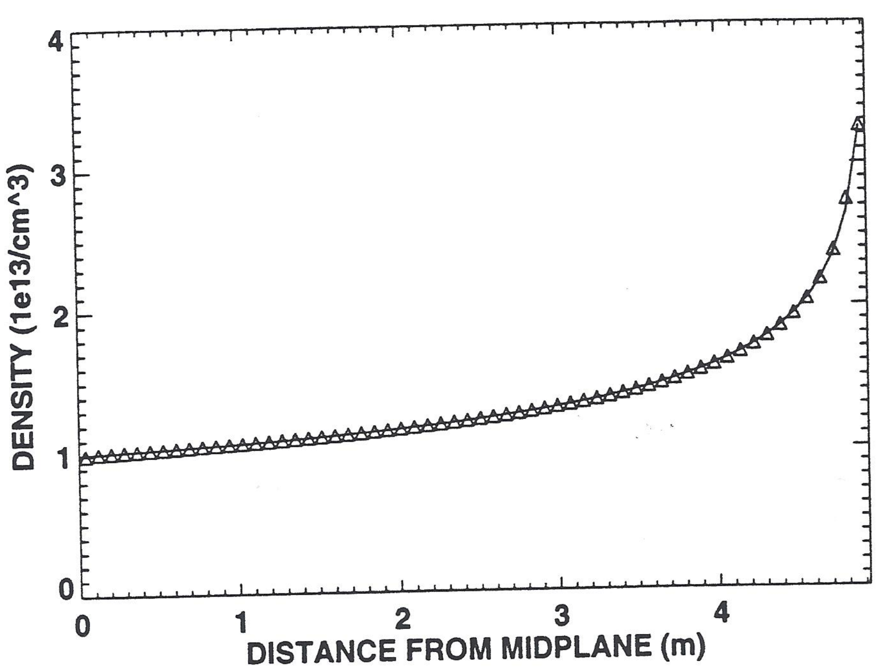
  FPET result: $e\Delta\phi_{\rm sh} = 4.9T_e(\text{wall})$

  classical result: $e\Delta\phi_{\rm sh} = 2.8T_e(\text{wall})$

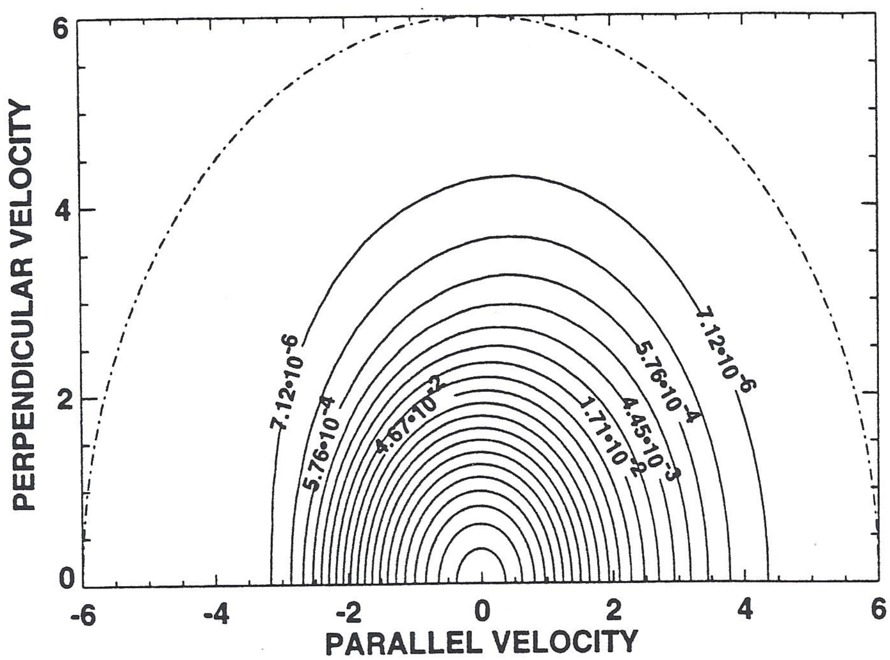
  (Note, classical sheath theory assumes Maxwellian electrons at the sheath edge.)

DIII-D H-MODE EXAMPLE



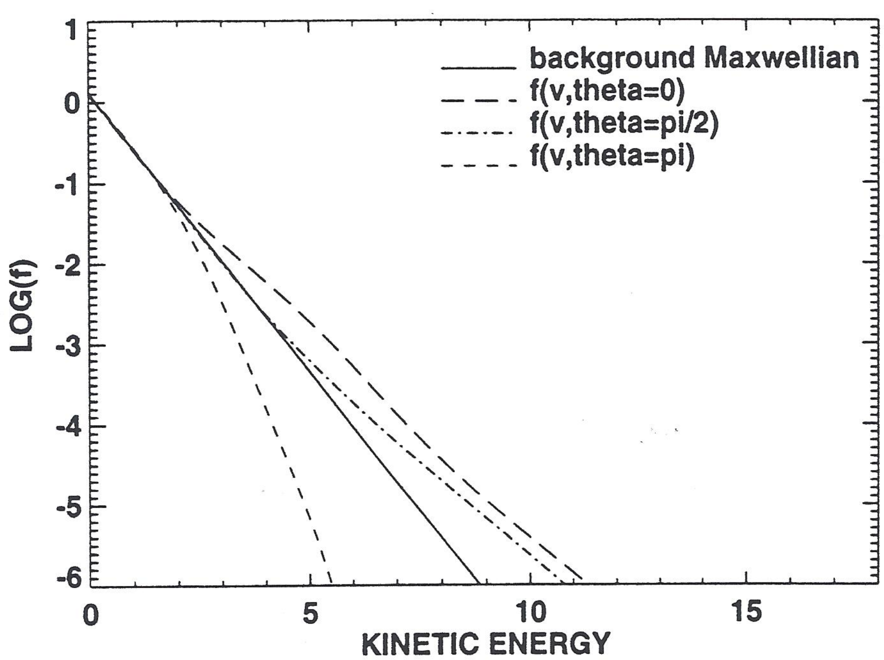


DIII-D H-MODE EXAMPLE

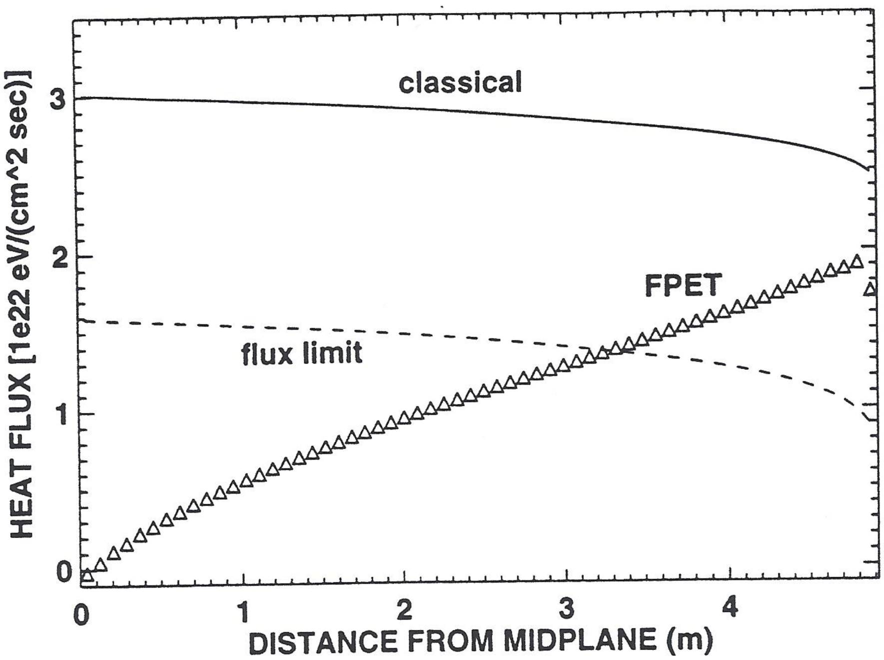


VELOCITY SPACE CUT AT 4 METERS FROM MID-PLANE

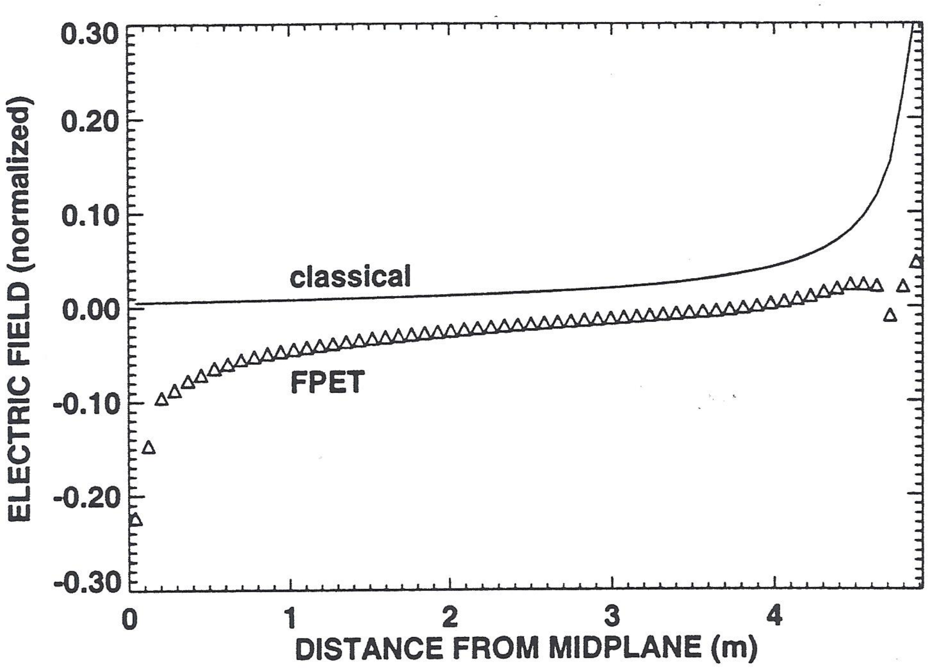
ALL VELOCITIES NORMALIZED TO THERMAL VELOCITY



DIII-D H-MODE EXAMPLE



FLUX LIMITING COEFFICIENT (above) = 0.15



Comparison with Classical Transport Theory

Transport Relations

Electric field —

$$eE_{\|} \;=\; p_e'/n_e \;+\; AT_e' \;+\; B\,(m_e/n_e\tau_e)\,\Gamma_e$$

Electron heat flux —

$$q_e \;=\; -C\,(n_e\tau_e/m_e)T_e\,T_e' \;+\; A\,T_e\,\Gamma_e$$

$$\tau_e = \frac{3\sqrt{m_e}}{4\sqrt{2\pi}\ln\Lambda e^4 Z_i}\,\frac{T_e^{3/2}}{n_e} \quad \text{and} \quad \Gamma_e = n_e\langle v_{\|}\rangle \quad .$$

Coefficients

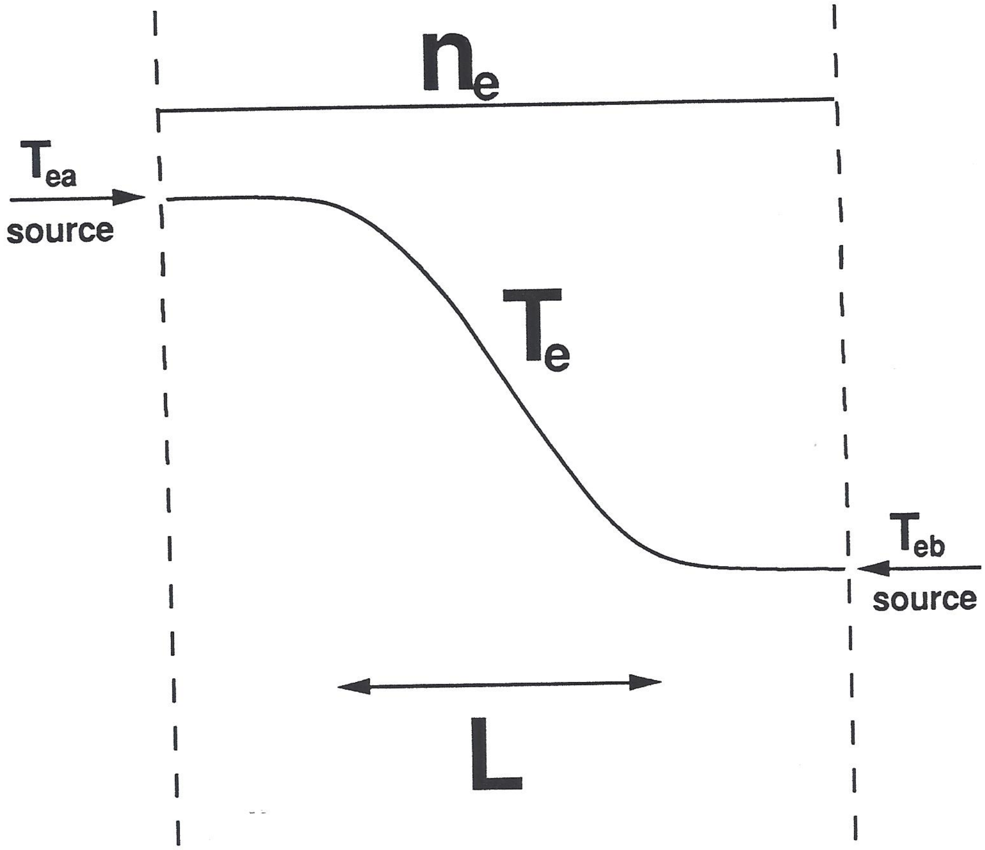
Braginskii ($Z_i = 1$) —

$$A = 0.71, \quad B = 0.51, \quad C = 3.16$$

Test particle operator ($Z_i = 1$) —

$$A = 1.10, \quad B = 1.18, \quad C = 2.38$$

Model Problem

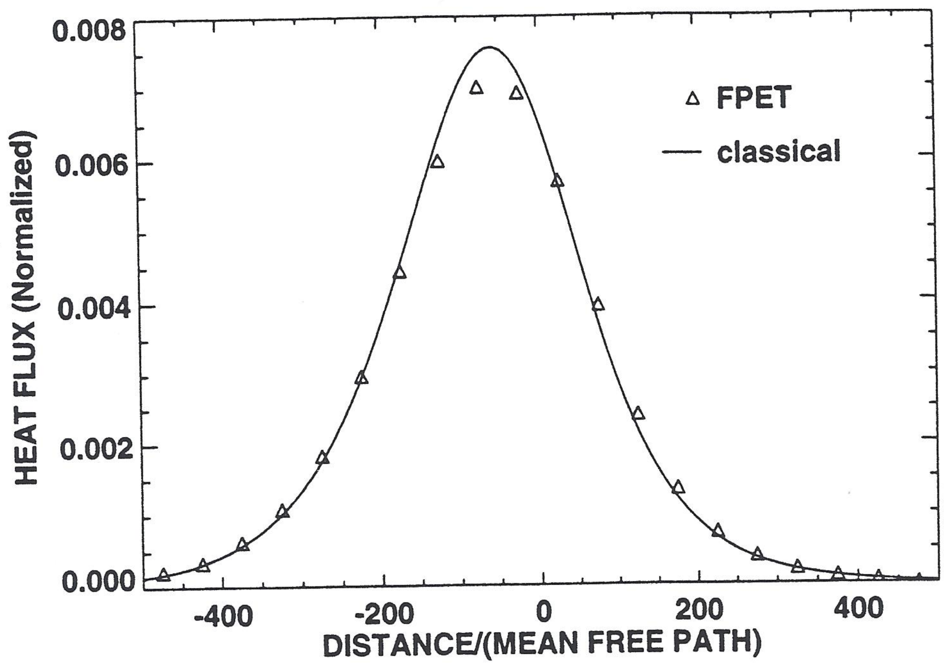


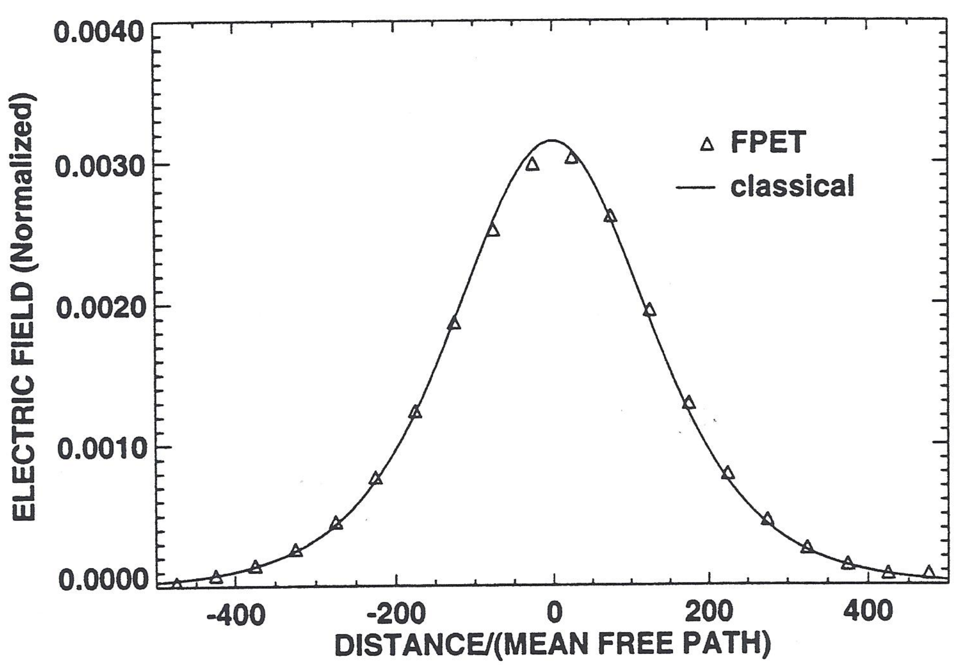
$$T_e = \frac{1}{2}\left[(T_{ea} + T_{eb}) + (T_{ea} - T_{eb}) \tanh\left(\frac{3z}{L}\right)\right]$$

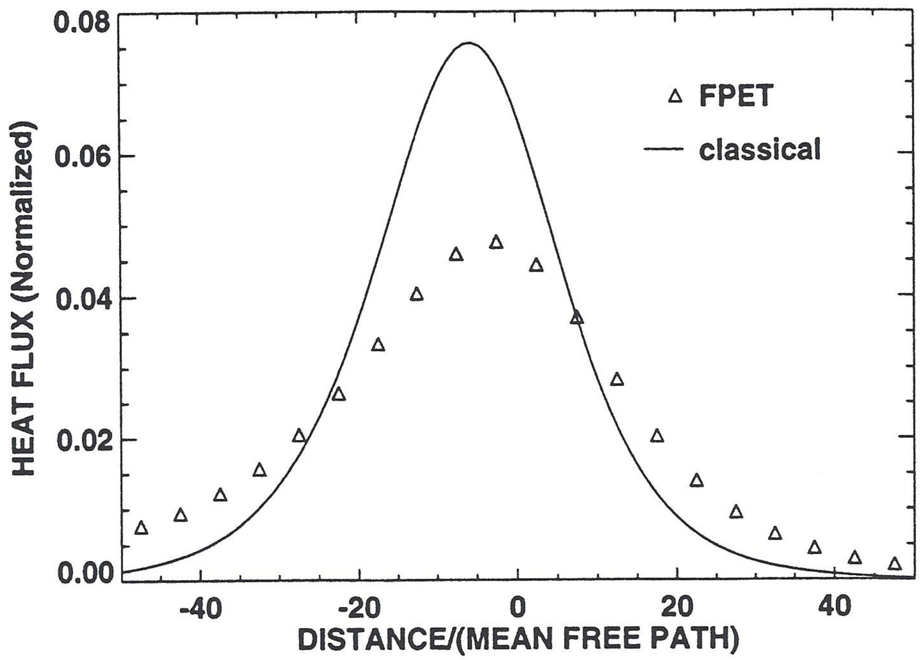
Three cases are shown in the following figures —

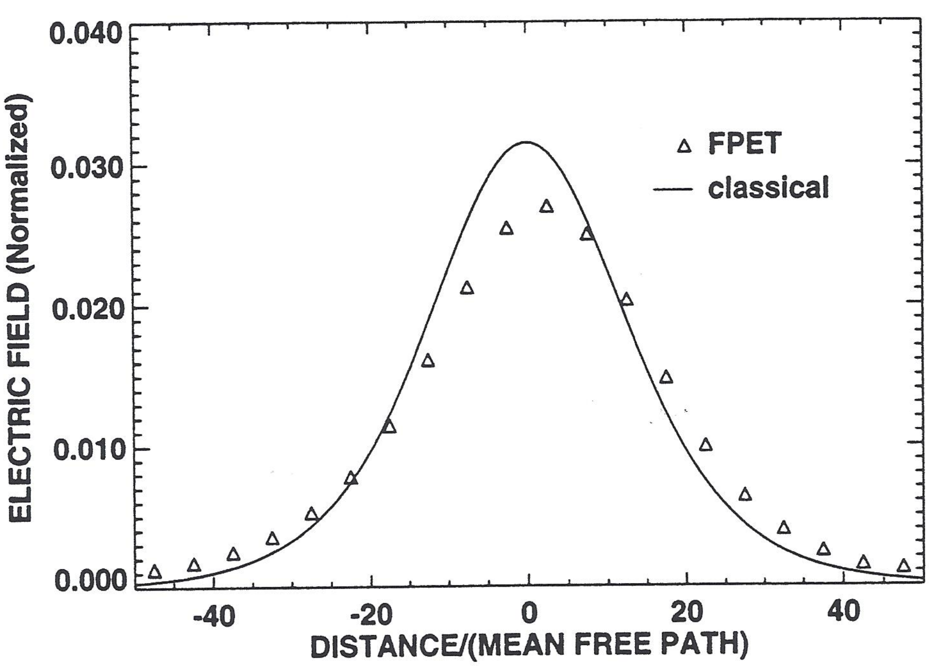
$$L/\lambda_{\mathrm{MFP}} = 500,\ 50,\ 15$$

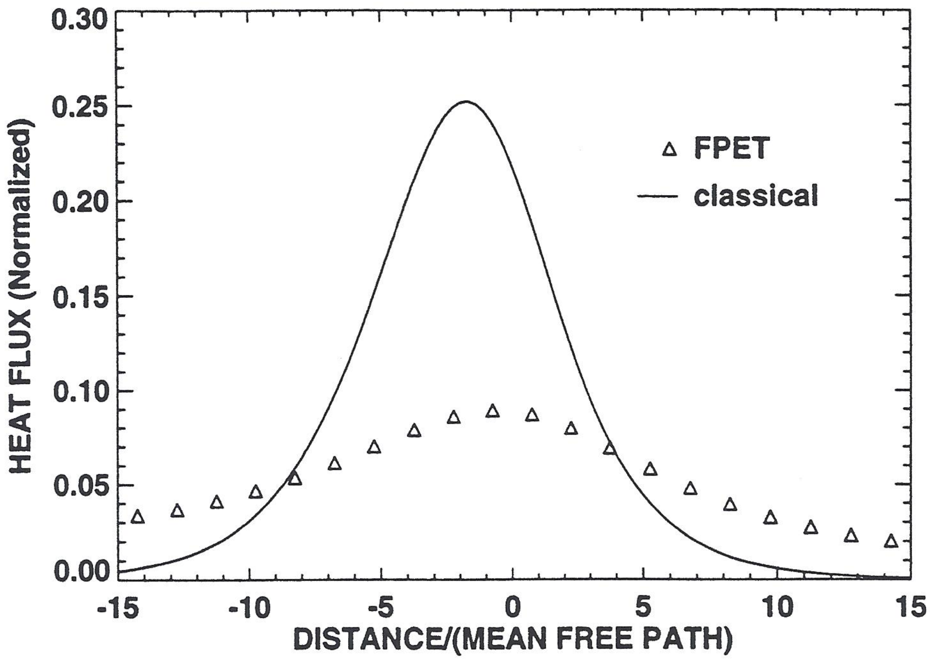
In all three cases $T_{eb} = T_{ea}/2$ and the electric field is determined by imposing the **zero flux** condition.

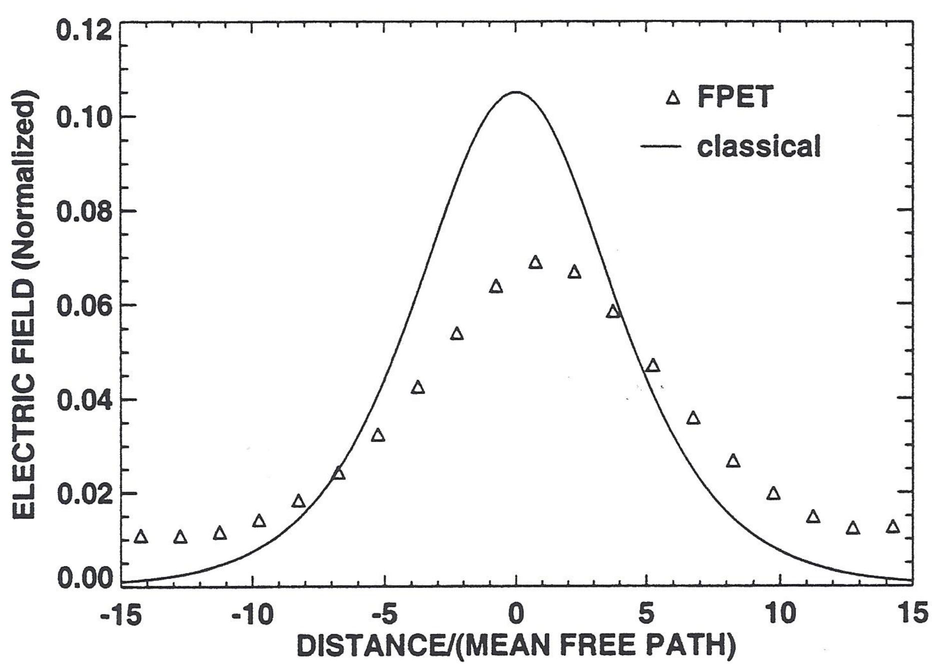

0.008
0.006
0.004
0.002
0.000
HEAT FLUX (Normalized)
FPET
classical
-400
-200
0
200
400
DISTANCE/(MEAN FREE PATH)


0.0040
0.0030
0.0020
0.0010
0.0000
ELECTRIC FIELD (Normalized)
FPET
classical
-400
-200
0
200
400
DISTANCE/(MEAN FREE PATH)


0.08
0.06
0.04
0.02
0.00
HEAT FLUX (Normalized)
FPET
classical
-40
-20
0
20
40
DISTANCE/(MEAN FREE PATH)


0.040
0.030
0.020
0.010
0.000
ELECTRIC FIELD (Normalized)
FPET
classical
-40
-20
0
20
40
DISTANCE/(MEAN FREE PATH)


0.30
0.25
0.20
0.15
0.10
0.05
0.00
HEAT FLUX (Normalized)
FPET
classical
-15
-10
-5
0
5
10
15
DISTANCE/(MEAN FREE PATH)


0.12
0.10
0.08
0.06
0.04
0.02
0.00
ELECTRIC FIELD (Normalized)
FPET
classical
-15
-10
-5
0
5
10
15
DISTANCE/(MEAN FREE PATH)